Abstract of the paper “Upgrading edges in the Maximal Covering Location Problem”

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The maximal covering location problem was first introduced by Church and ReVelle (1974). This problem consists on locating a fixed number of facilities maximizing the covered demand of a given set of clients. A client is hereby considered to be covered if their distance to a facility is smaller than or equal to a given coverage radius. Since its origins, this model has been widely studied in the literature under different perspectives, as for example, the solution domain of the problem (continuous, discrete, or on networks), the coverage assumptions (gradual coverage, cooperative coverage, etc.), uncertainty on the model parameters.

Common to all those problems is, however, that the input parameters of the network and the problem are not decision variables of the model. In this work, we propose a different approach dealing with the maximal covering location problem on networks assuming that edges can be *upgraded*. Upgrading an edge means reducing its length considering *i)* a cost per unit of reduction, *ii)* an upper bound of reduction in each edge, and *iii)* a budget constraint on the overall cost of reduction.

Baldomero-Naranjo et al. (2022) studies this upgrading version of the maximal covering location problem with edge length modifications on networks. This problem aims at locating p facilities on the vertices (of the network) so as to maximise coverage, considering that the length of the edges can be reduced at a cost, subject to a given budget. Hence, we have to decide on: the optimal location of p facilities and the optimal edge length reductions.

State-of-the-art

Next, we briefly review the literature of upgrading problems. The upgrading version of many classical problems has been studied during the last decades, e.g. for the spanning tree problem (Álvarez-Miranda and Sinnl, 2017), for the min-max spanning tree problem (Sepasian and Monabbati, 2017), for bottleneck problems (Burkard et al., 2004), for minimum flow cost problems (Demgensky et al., 2002), for the shortest path problem (Dilkina et al., 2011), for the maximal shortest path interdiction problem (Zhang et al., 2021), or for communication and signal flow problems (Paik and Sahni, 1995).

In the context of node upgrading location problems (the weight of the vertices can be modified subject to a prespecified budget), the following problems have been analysed: the 1-median problem (Gassner, 2007), the 1-center problem (Gassner, 2009), the Euclidean 1-median problem (Plastria, 2016), the p-median problem (Sepasian and Rahbarnia, 2015), and the hub-location problem (Blanco and Marín, 2019), among others.

In the context of edge upgrading location problems, we are aware of only two directly related publications: upgrading the 1-center problem (Sepasian, 2018) and upgrading the obnoxious p-median problem on trees (Afrashteh et al., 2020).

Therefore, the main aim of this paper is to fill this gap in the literature by studying the upgrading maximal covering location problem with variable edge lengths.

For sake of clarity, we summarise the cited literature of upgrading problems in Table 1. We add two columns labelled N and C for network and continuous problems.

Field	Upgrading	N	C	Problem and reference
Location problems	Nodes	X		1-center: Gassner (2009).
		X		Hub-location: Blanco and Marín (2019).
		X		1-median: Gassner (2007).
			X	Euclidean 1-median: Plastria (2016).
		X		p-median: Sepasian and Rahbarnia (2015).
	Arcs/edges	X		1-center: Sepasian (2018).
		X		Maximal covering: our paper.
		X		Obnoxious p-median: Afrashteh et al. (2020).
Others	Nodes	X		Communication and signal flow problems: Paik and Sahni (1995).
		X		Shortest path: Dilkina et al. (2011).
		X		Spanning tree: Álvarez-Miranda and Sinnl (2017).
	Arcs/edges	X		Maximal shortest path interdiction problem: Zhang et al. (2021).
		X		Min–max spanning tree: Sepasian and Monabbati (2017).
		X		Minimum flow cost: Demgensky et al. (2002).

Table 1: Summary of literature review of upgrading problems.

Contribution

In this paper, we have tackled an interesting problem: the upgrading maximal covering location problem with edge length modifications, Up-MCLP. As far as we know, it is the first time that this problem is discussed in the literature.

Since we were dealing with a new problem, we have proposed three different mixed-integer formulations to model the situation from various perspectives:

- The first formulation for the problem, (Flow-Cov), models the paths using flow variables and includes assignment variables to represent which facility covers the demand of each node.
- The idea of the second formulation, (Path), is to model the path through the immediate successor.

- Finally, the third formulation, (Path-Cov), merges components from the first formulation with the second formulation, i.e., this formulation models the path through the immediate successor and includes assignment variables.

Furthermore, using the intrinsic properties of the problem, we develop an effective preprocessing phase that allows us to reduce the dimension of the proposed formulations, allowing us to solve instances faster and also solve larger instances than without preprocessing. This procedure fixed many variables and reduced the size of the problem considerably. Moreover, we present several theoretical results that improve the proposed formulations, proving that the integrality condition on some variables can be relaxed and strengthening some families of constraints. Besides, exploiting the structure of the problem and analysing the proposed model, we derive several sets of valid inequalities that eliminate symmetries and even further improve the solution times of the formulations. Furthermore, we solve the separation problem for the families of valid inequalities whose cardinality is exponential.

The performance of the three formulations and the improvement provided by the preprocessing phase and the valid inequalities can be appreciated in the computational results included in the paper. In these experiments, it can be seen that the most efficient formulations for solving Up-MCLP are the (Flow-Cov) and the (Path-Cov). In complete graphs, there is little difference in performance between these two formulations, while in sparse graphs the (Path-Cov) formulation performs better than the (Flow-Cov) one. In both types of graphs, the addition of valid inequalities allows us to optimally solve a larger number of instances within the time limit.

We believe that this work could be an encouraging starting point to address the upgrading version of other classical location problems. This is especially interesting since the development of the upgrading version of location problems is a growing area, in which there are many open questions.

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